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Protective Capital and Poverty Traps

Dr. Ali Saedvandi¹

Abstract

We investigate a protective role for capital and the implications on poverty reduction policies. Within the framework of an overlapping generations model, the accumulation of tax-funded 'protective capital' increases the survival probability of productive capital from disastrous events. Key findings highlight how a low survival probability of capital can stimulate a shift to higher or lower early consumption, which might result in escapable or inescapable traps. The model explains that injection of capital can be ineffective if the required level of protection has not been attained. We show how higher TFP assists wage tax system to escape from traps.

JEL: I32, O19, O41

Keywords: Growth, Poverty Trap, Overlapping Generations Model, Depreciation

1. INTRODUCTION

Disasters, natural or human-made, can take place anywhere in the world regardless of the income level of the people affected. Developed nations can incur large amounts of damages from catastrophic events in absolute terms; however, less developed countries witness more destructive impacts from disasters on the lives of inhabitants, particularly when losses are compared with their income level.

The fact is that people of the less developed countries (LDCs)—which are very likely also highly indebted poor countries (HIPCs) in Sub-Saharan Africa, Latin America, and Asia—are more vulnerable to natural disasters. According to a United Nations' report (2008), the frequency of disasters per year increased from 78 to 351 during 1970-2006. As a matter of fact, the average number of casualties for each disaster has been on a declining trend. In contrast, the total number of people affected by natural hazards and the estimated costs of damages has sharply increased. The UN report concludes, 'As disasters have become less life-threatening, they have become more threatening to the well-being of the communities that are hit' (Page 81).

The current work attempts to elucidate a relationship between disastrous events and the possibility of the occurrence of poverty traps. The paper suggests that calamities can produce rather short-term damages in developed nations, contrasting with the long-term catastrophic consequences in LDCs and HIPCs, because of the magnitude of losses relative to the GDP in poorer nations. This notion can be supported by the data on the estimated cost of the worst damages relative to income in the past 30 years. According to United Nations (2008), while the 1980 earthquake in Italy, among OECD countries, is recorded as the costliest natural event (4.3

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percent of the GDP) in the last 50 years, damages in less developed nations can amount to multiple times of their income; for instance, a loss of 309 percent of the GDP in North Korea in 2005.

Our model shows that poor societies are unable to fend off the catastrophic impacts of disastrous events on their productive capital unless some particular level of protective capital has already been accumulated. One negative consequence of this issue is that LDCs that obtain lesser amounts of protective capital than the required threshold level are susceptible to being entrapped in poverty in an inescapable steady-state, which might render traditional poverty reduction policies ineffectual.

An extensive body of literature exists on the causes of the poverty trap. Azariadis (2006) categorizes theories in this field into three groups: threshold effects, institutions, and neighbourhood effects. This paper develops a 'threshold effect model' based on the endogenous lifetime model Chakraborty (2004) and its revised version published in *Poverty Trap* (2006). Chakraborty endogenizes longevity through the probability of the survival of individuals from the first period to the next in an overlapping generations model. The probability of survival varies with the level of tax-funded health capital. Chakraborty finds supportive evidence for his hypothesis from an empirical analysis of the relationship between life expectancy and long-term economic growth.

The present paper attempts to endogenize depreciation of physical capital through an overlapping generations model which includes a probability function for the survival of productive capital. The probability that capital will remain unscathed from the first period of an individual's life to the next depends upon the accumulated level of tax-funded protective capital. By modelling the probability of survival, this paper aims to account for different reactions to the possibility of catastrophic events from economies which are at different stages of development. The model predicts a low level of either productive or protective capital would result in stagnation.

In contrast to Chakraborty (2004), we suppose constant longevity that eliminates an impact on the propensity to save, but introduces a new element because the propensity to save depends on the remuneration of saving. It might be interpreted that one foundation of endogeneity of the saving rate is replaced by another one; however, this fact did not result in poverty trap when the Chakraborty's logarithmic utility function is investigated. The main contribution of our work is the unique methodology to analyze the implication of CRRA utility into the theory of poverty trap and the role of 'intertemporal elasticity of substitution' in emergence of an escapable or an inescapable trap.

Although disasters are rarely associated with poverty traps in the theory of growth, the concept of rare disasters has been used by Rietz (1988) and Barro (2006) to explain the equity premium puzzle. Barro (2006) employs a stochastic model to calibrate the equity premium. Our deterministic model can be viewed as the first step into a more general stochastic model.

The paper proceeds as follows. Section II further distinguishes between protective capital and production capital and explains its importance in the set up of the model. In section III, different types of poverty traps are defined. The model is defined in section IV, which examines the effects of protective capital on capital accumulation in an OLG model. The role of two different shapes of productive capital survival probability functions, pulse and general form, on the behaviour of the model are also discussed in this section. Next, some policies on poverty reduction, such as external

capital injection and wage taxation system, are discussed in section V. The paper concludes in section VI.

2. PROTECTIVE VERSUS PRODUCTIVE CAPITAL

In the last half of the previous century, the theory of growth has persuasively emphasized the role of the accumulation of productive capital as the explanatory cause of development. One strand of models identifies different types of capital such as physical, human, and social capital as stimulants of growth. The common premise of these models is that the role of capital in an economy is solely to be productive. The second branch focuses on productivity itself. These models associate the observed ongoing, widespread, and growing inequality in standards of living within and among nations with the variety of productivity levels observed around the globe.

It is conventionally believed that in addition to the levels of different types of capital and the observed varieties of productivities, some other still unknown factors are involved in accounting for the magnitude of per capita income gap around the world. The proposed model introduces a new function for capital which is called protective capital. Protective capital preserves productive capital against hazards. Natural calamities such as droughts, tsunamis, earthquakes, floods, landslides, hurricanes, epidemics, and volcanoes as well as man-made disasters such as depressions, credit crunches, industrial events, transportation accidents, wars, ethnic tensions, social or political unrests, pollution, and climate change all endanger the quality of human life. The question is how the impacts of some catastrophic events with the same magnitude drastically vary across nations. Why does an earthquake claim more than 100,000 lives in Haiti and still remarkable stronger one in Chile leaves less than 500 victims? (Wikipedia)

This paper argues that a high level of protective capital is the main contributor to the relatively restricted damage of an earthquake in one of the two countries. Thus, a direct upshot of the recognition of the role of protective capital is to increase its investment level.

When it comes to funding protective capital, its stark distinction from productive capital appears. Private investors look at the expected rate of return for any investment to decide whether to invest in productive capital. Therefore, investors continue funding productive capital as long as the expected return meets the investors' required return. However, for protective capital, the concept of rate of return is less clear and depends upon the expected loss of each catastrophic event as well as the expected return of productive capital. In addition, protection usually requires collective actions and market failure can be a consequence of investing in protective capital only through free markets. Therefore, in the present model productive capital is funded through free market in contrast with protective capital that is funded by levying wage tax.

This setting resembles Chakrobarty's model with health capital; however, here we consider a minimal role for protective capital. Protective capital only boosts the expected return of capital and does not change the longevity of individuals. Our objective is to present that the impact of protective capital accumulation on the individuals' expectation can be a sufficient condition for escapable or inescapable poverty traps, depending on individuals' preferences.

3. TYPES OF POVERTY TRAP

The classical definition of poverty trap is 'any self-reinforcing mechanism which causes poverty to persist' Azariadis & Stachurski (2005). A more technical description is that a poverty trap occurs when, in a multi-equilibria growth model, the mechanisms of free markets would be unable to move the economy from a low-income steady-state to any of high-income steady-states without an intervention from external sources or from government. In this line, we define three types of poverty trap: (1) inescapable poverty trap, (2) alternating poverty trap, and (3) semi-trap.

Inescapable poverty trap: in the framework of the proposed model, this is a type of poverty trap that external capital injection would be ineffective for escaping from the trap. In other words, this is a case where no amount of foreign financial aid would be able to lead the economy towards the other high-income steady-state level(s). In turn, if capital injection would possibly be effective in poverty reduction in a poverty trap, this is called an 'escapable trap'.

Alternating poverty trap: An alternating trap occurs when an economy alternates between two inferior and superior income levels. Neither of the two levels are steady-states, though the combination of the two can be observed as a mixed equilibrium.

Semi-trap: it is a steady-state in a multi-equilibria free-market economy, which the economy is able to surpass the low-income but never reaches the potentially high-income steady-state and will be entrapped in an equilibrium between the two extreme levels.

4. PROTECTIVE CAPITAL AND OLG MODELS

We assume a standard Diamond-type two-period overlapping generations model (Diamond, 1965). Individuals save a portion of their wage during the first period in hopes of consuming the entirety of the invested funds plus the returns from capital market in the second stage of their life. Yet, there is a possibility that at the end of the first period investors could lose their entire investment due to a disastrous event. The probability of the loss depends upon the stock of tax-funded protective capital.

A. The Environment

A constant number of individuals cohorts, $v = 0, 1, 2, \dots$ are born during each period and inelastically supply their labour force for only one period in exchange for a wage, w_t . Individuals of each cohort live for two periods and during the second period of their life supply no labour force, but consume their savings plus returns from capital market.

In addition to productive capital, the economy accumulates protective capital, which increases the probability of the survival of productive capital. We assume that at the end of the first period of the cohort's life a major disastrous event takes place. In the absence of protective capital, a low portion of productive capital would remain for the second period of individuals' lives. Therefore, in addition to the traditional depreciation rate of productive capital, it is also important to define $\pi_{(q)}: R^+$

$\rightarrow [P_{min}, P_{max}]$ such that $0 \leq P_{min} < P_{max} \leq 1$, as a probability function of productive capital survival when the protective capital level is $q_t \geq 0$. With no investment in protective capital, there is only a small chance of productive capital survival. This low probability of survival is denoted by P_{min} . In turn, as protective capital is accumulated over time, the probability of productive capital survival converges towards an upper

limit P_{max} . Accordingly, disastrous events are expected to destroy $1 - \pi_{(q)}$ of the entire stock of productive capital.

The probability function, $\pi_{(q)}$, is increasing in the stock of protective capital, q , and satisfies

$$\lim_{q \rightarrow 0} \pi_{(q)} = P_{min} \geq 0 ,$$

Assumption (2)

$$\lim_{q \rightarrow \infty} \pi_{(q)} = P_{max} \leq 1 ,$$

Assumption (3)

$$\pi'_{(q)} \geq 0^2 .$$

Assumption (4)

Individuals maximize their expected CRRA utility function over the two periods:

$$U_t = (c_t^{1-\frac{1}{\theta}} - 1) / (1 - \frac{1}{\theta}) + \pi_{(q)} (c_{t+1}^{1-\frac{1}{\theta}} - 1) / (1 - \frac{1}{\theta}) , \quad (1)$$

subject to two budget constraints:

$$c_t^t \leq (1 - \tau_t)w_t - s_t ,$$

$$c_{t+1}^t \leq \tilde{r}_{t+1}s_t ,$$

where $\theta > 0$. Moreover, s_t is the amount of savings at time t and \tilde{r}_{t+1} is the expected total return from the capital market on saving s_t .

The production function $F(K, L) = AK^\alpha L^{1-\alpha}$ is Cobb-Douglas with constant returns to scale, capital shares of $\alpha \in (0, 1)$, and $A > 0$. Therefore, in per worker terms

$$f(k) = Ak^\alpha , \quad (2)$$

where k_t stands for per worker productive capital. We also assume:

$$\alpha + \theta > 1$$

Assumption (1)

In a competitive free market, wage and capital rent rates are determined as follows:

$$w_t = (1 - \alpha)Ak_t^\alpha , \quad (3)$$

$$R_t = 1 + \alpha Ak_t^{\alpha-1} - \delta_k , \quad (4)$$

where $\delta_k \in [0, 1]$ is the depreciation rate of productive capital. Also, since each period lasts for half of a cohort's life, it can be assumed that $\delta_k = 1$.

Therefore, the expected return from the capital market, \tilde{r}_{t+1} , is the gross return from the capital market, R_t , multiplied by the chance of survival, $\pi_{(q)}$.

$$\tilde{r}_{t+1} = R_{t+1} \times \pi_q .$$

Despite the fact that disastrous events are systematic, undiversified risks in the economy, we assume that each country takes actuarially fair insurance from international institutions and the size of the economy and the damages from disasters are such small and uncorrelated with the rest of the world that the international institutions do not require risk premium. This assumption makes π_q deterministic. Also, Blanchard (1985) and Yari (1956) are followed to abstract from the risk associated with the expected return, R_{t+1} . In fact, we assume perfect annuities system with intermediation of mutual funds. These international insurance setup and annuities system guarantee the deterministic \tilde{r}_{t+1} , which differs among countries depending upon the values of $\pi_{(q)}$.

Protective capital is accumulated by the public sector through a flat wage tax rate $\tau \in [0, 1]$ and depreciates at a constant rate $\delta_q \in [0, 1]$.

The law of motion for protective capital is

$$q_{t+1} = \tau w_t + (1 - \delta_q)q_t .$$

for all cohorts $t \geq 0$ with $q_0 > 0$ as given.

Similar to δ_k , we assume $\delta_q = 1$:

² Two functional forms that satisfy the properties of the probability function are $\pi_{(q)} = \beta q / (1 + q)$ where $\beta \in [0, 1]$ and $\pi_{(q)} = \beta (1 - a)^q$ where $a > 0$.

$$q_{t+1} = \tau w_t. \quad (5)$$

Notably, the probability of disasters is unaffected by the value of q_t and q_t only impacts on the size of losses from a disaster.

B. General Equilibrium

Given the wage and capital rent rates and the deterministic structure of rate of return, individuals' maximization process of their CRRA utility function subject to the budget constraints yields Euler's equation:

$$\frac{c_{t+1}}{c_t} = (\pi_{(q)} \tilde{r}_{t+1})^\theta. \quad (6)$$

The above equation shows how economies with different 'intertemporal elasticity of substitution', (θ) respond to a change in expected return of capital, \tilde{r}_{t+1} .

Then equations (3), (4), and (6) yield the saving function:

$$s_t = \frac{1}{1 + \pi_{(q)} \tilde{r}_{t+1}^{1-\theta}} (1 - \tau) w_t = \frac{1}{1 + \pi_{(q)} (\alpha A k_{t+1}^{\alpha-1})^{1-\theta}} (1 - \tau) (1 - \alpha) A k_t^\alpha \quad (7)$$

The capital stock in period $t + 1$ would be equal to the level of savings in period t :

$$k_{t+1} = \frac{1}{1 + \pi_{(q)} (\alpha A k_{t+1}^{\alpha-1})^{1-\theta}} (1 - \alpha) (1 - \tau) A k_t^\alpha \quad (8)$$

Then we have:

$$k_{t+1} (1 + \pi_{(q)} (\alpha A k_{t+1}^{\alpha-1})^{1-\theta}) = (1 - \alpha) (1 - \tau) A k_t^\alpha \quad (9)$$

One obvious steady state solution for equation (8) is $k = 0$; however, there is no closed form solution for the other possible steady states. As follows we introduce a dynamic analysis for the other unique stable steady state, based on the method of Barro and Sala-i-Martin (2004). Thus we define average productivity of capital $f_t \equiv A \bar{k}^{\alpha-1}$ and rewrite equation (9) at the steady-state level:

$$1 + \pi_{(q)} [\alpha \bar{f}]^{1-\theta} = (1 - \alpha) (1 - \tau) \bar{f} \quad (10)$$

The above transformation towards average productivity of capital facilitates the analysis of steady state solutions. Also, it is customary to analyze the dynamics of Diamond-type models by describing the properties of the law of motion of the capital k . Since it is impossible to rearrange equation (9) in the form of $k_{t+1} = J(k_t)$, there is no exact analysis for CRRA models. In the following we attempt to offer a reasonable approximation which enables us to analyse the dynamics of CRRA models.

Although the LHS of equation (10) does hold for all the values of capital, the RHS is only valid for the steady state levels. The general form of RHS for all levels of capital is $(1 - \alpha) (1 - \tau) A k_t^\alpha / k_{t+1} \leq (1 - \alpha) (1 - \tau) A f_t$. Nevertheless, the following equation suffices for dynamic analysis by adopting the linear approximation:

$$1 + \pi_{(q)} [\alpha f_t]^{1-\theta} = (1 - \alpha) (1 - \tau) f_t \quad (11)$$

In order to investigate the role of protective capital on the possibility of the poverty trap, we assume a pulse function for the probability of productive capital survival.

C. Pulse Probability Function

Initially, it may be supposed that the survival probability of productive capital is a pulse function. In other words, there is a threshold, Q , for protective capital to be effective:

$$\pi_{(q)} = \begin{cases} P_{max} & \text{if } q \geq Q \\ P_{min} & \text{if } q < Q \end{cases}.$$

In the case of $q < Q$, the probability of the survival of productive capital from disastrous events is low ($\pi_{(q)} = P_{min}$). In this circumstance, depending upon the economy's utility function, young individuals prefer to consume more or less of their

wages immediately. As $\pi_{(q)} = P_{min}$, the model converges to a standard CRRA-Cobb Douglas OLG model with a unique stable steady-state level.

Accordingly, it may be further supposed that \bar{k}_{min} represents the steady-state level of k_t i.e. $k_{t+1} = k_t = \bar{k}_{min}$. The average productivity of capital at steady-state level, \bar{k}_{min} , may be defined as $f_{min}^* \equiv A \cdot \bar{k}_{min}^{\alpha-1}$ and equation (8) at the steady-state level can be rewritten:

$$1 + P_{min}[\alpha f_{min}^*]^{\frac{\theta-1}{\theta}} = (1 - \alpha)(1 - \tau)f_{min}^* . \quad (9)$$

The same analogy can be employed in the case of $\pi_{(q)} = P_{max}$:

$$1 + \mu^{\frac{-1}{\theta}}[\alpha f_{max}^* P_{max}]^{\frac{\theta-1}{\theta}} = (1 - \alpha)(1 - \tau)f_{max}^* . \quad (10)$$

The left hand side of equations (9) and (10) are defined as $\varphi \equiv 1 + \mu^{\frac{-1}{\theta}}[\alpha f P]^{\frac{\theta-1}{\theta}}$ and the right-hand side as $L \equiv (1 - \alpha)(1 - \tau)f$; both can take indices of *max* or *min*. FIGURES 1, 2, and 3 illustrate equations (9) and (10) when θ is less than, greater than, or equal to 1, respectively. The RHS's of equations (9) and (10) are the same line ($L_{max} = L_{min} = L$), which is a straight line with the slope of $(1 - \alpha)(1 - \tau)$ and goes through the origin. A social planner might reach the required outcome by changing the slope of L through fine tuning of tax rate τ .

If the stock of capital in an economy is below the steady-state level, $k_t < \bar{k}$, then with the accumulation of capital the value of k_t approaches to \bar{k} ; and equivalently the value of average productivity of capital, f_t , decreases from infinity towards f^* . This is presented in FIGURES 1, 2, and 3, where due to a decrease in f_t , the values of L decreases and the values of φ may increase, decrease, or remain constant depending upon θ . No matter what the value of θ is, the gap between L and φ decreases as more capital accumulates. Similar logic is valid when the initial capital stock is above the steady-state level, $k_t > \bar{k}$.

In the discussion that follows, the possibility of poverty traps are investigated and an attempt is made to analyze policies to avoid or escape the trap. It is noteworthy that all of the analyses are based on the values of f_t instead of k_t . Care must be taken regarding the inverse relation between the two values: as capital stock, k_t , goes to infinity, the average productivity of capital, f_t , goes to zero. Furthermore, whenever the two values of k_t are compared, the higher (lower) k_t is associated with the lower (higher) f_t .

Two parameters have more significant impacts on the behaviour of the model: (1) the intertemporal elasticity of substitution, $1/\theta$; and (2) the protective capital threshold level, Q .

As mentioned, the shapes of φ_{min} and φ_{max} depend upon the value of θ . As θ is equal to, less than, or greater than 1, the shape of φ would be a straight line, or a decreasing convex, or an increasing concave curve. In any event, the equations (9) and (10) have unique stable steady-state solutions which result from the intersection of functions L and φ . Barro & Sala-i-Martin (2004).

The inverse value of intertemporal elasticity of substitution, θ , determines which steady-state level, f_{min}^* or f_{max}^* , would be higher. In the case of $\theta < 1$, the curve of φ_{max} is below the curve of φ_{min} and both are decreasing convex curves (FIGURE 1). Therefore, φ_{min} intersects L first at a higher f^* . Thus, $f_{max}^* < f_{min}^*$ which is equivalent to $\bar{k}_{max} > \bar{k}_{min}$. In such an economy, as the capital survival probability increases, young individuals favour higher savings over early consumption, which results in lower consumption in the first period of their lives and higher consumption in the second period ($s_\pi > 0$).

<FIGURE 1>

On the contrary, if $\theta > 1$, the curve of φ_{max} is above φ_{min} and both are increasing concave curves (FIGURE 2). Therefore, φ_{max} intersects L first at a higher f^* . Thus $f_{min}^* < f_{max}^*$, which is equivalent to $\bar{k}_{min} > \bar{k}_{max}$. In such a situation, the elasticity of the saving function with respect to the probability of capital survival becomes negative ($s_\pi < 0$), and income effects dominate substitution effects. Therefore, individuals are less willing to postpone early consumption to the next period.

<FIGURE 2>

Another case occurs when the utility function is logarithmic ($\theta = 1$). Individuals with logarithmic utility functions do not vary their investment decisions with the changes in the expected rate of return or the probability of capital survival. As FIGURE 3 illustrates in such cases, both curves of φ_{max} and φ_{min} collapse into one single straight line $\varphi_{max} = \varphi_{min} = 1 + \mu^{-1/\theta}$.

<FIGURE 3>

A poverty trap might occur depending upon the value of the threshold level of protective capital Q relative to the steady-state levels \bar{k}_{max} and \bar{k}_{min} . In order to facilitate the current analysis of poverty trap based on the values of f_t , an equivalent value for Q is defined. From equation 5, protective capital in the second period is fully funded through a given wage tax rate:

$$q_{t+1} = \tau(1 - \alpha)Ak_t^\alpha. \quad (11)$$

Then, the minimum productive capital that is required to raise the level of protective capital to above the threshold level Q is

$$k_Q = (\tau(1 - \alpha)/Q)^{1/\alpha}. \quad (12)$$

Similar to f_{max}^* and f_{min}^* in equations (9) and (10), $f_Q \equiv A.k_Q^{\alpha-1} = A^{1/\alpha} \cdot (\tau(1 - \alpha)/Q)^{(1-\alpha)/\alpha}$. This is the equivalent value of f^* that is associated with the threshold level Q .

Depending upon the relative value of f_Q to f_{max}^* and f_{min}^* , and for the cases wherein θ is less than, greater than, or equal to 1, an economy can be in no trap, in an escapable, or an inescapable trap, or even in alternating traps. Propositions 1 to 5 further distinguish these different possibilities.

Proposition 1. In a CRRA-Cobb Douglas OLG model with a pulse probability function of productive capital survival, a poverty trap does not exist if $f_Q > \max(f_{min}^*, f_{max}^*)$.

Proof: If $\theta < 1$, then $f_{min}^* > f_{max}^*$ (FIGURE 1, Panel (a)). The first situation is that the economy starts from a stock of productive capital level, k_t , for which $k_t < k_Q < \bar{k}_{min} < \bar{k}_{max}$ (equivalently $f_t > f_Q > f_{min}^* > f_{max}^*$). Since $k_t < k_Q$, protection is not effective yet and the capital stock grows on a path towards \bar{k}_{min} , or equivalently average productivity of capital decreases towards f_{min}^* . However, the value of k_t will reach k_Q before \bar{k}_{min} , and at this level of capital, protection is activated, which jumps to the maximum probability of capital survival. In short, the path towards steady-state shifts from φ_{min} to φ_{max} . Higher capital survival probability implies a higher expected rate of return for capital, and, since $\theta < 1$, substitution effects dominates income

effects and the economy tend more towards saving. Capital is accumulated on a new path towards a new steady states level of \bar{k}_{max} (f_{max}^*) and surpasses \bar{k}_{min} (f_{min}^*) without being entrapped.

If $\theta > 1$, then $f_{max}^* > f_{min}^*$ (FIGURE 2, Panel (a)). It can be supposed that the economy starts from a stock of productive capital level k_t , in which $k_t < k_Q < \bar{k}_{max} < \bar{k}_{min}$ (equivalently $f_t > f_Q > f_{max}^* > f_{min}^*$). Again, since $k_t < k_Q$, protection is not effective yet and the capital stock grows on the path towards \bar{k}_{min} , but reaches k_Q earlier and jumps to the new path towards \bar{k}_{max} . The difference between this case and the case when $\theta < 1$ is that now income effects dominate substitution effects. In such an economy, as higher protection of capital is activated (k_t reaches k_Q), individuals increase their early consumption. Notably, the steady-state capital stock at the maximum probability of capital survival is lower than the steady state at the minimum probability of capital survival ($\bar{k}_{max} < \bar{k}_{min}$) because of individuals preferences for early consumption.

In both cases of θ , if $k_t > k_Q$, then the economy is on the path of φ_{max} and will move towards \bar{k}_{max} with no trap. ■

Proposition 2. In a CRRA-Cobb Douglas OLG model with a pulse probability function of productive capital survival, an inescapable poverty trap exists if $f_Q < \min(f_{min}^*, f_{max}^*)$.

Proof: If the initial level of capital stock is below \bar{k}_{min} , then the capital stock converges towards \bar{k}_{min} without hitting k_Q . We suppose that a remarkable measure of foreign aid is injected into the economy increasing the level of capital stock above the threshold level; $k_t > k_Q$. The proposed model will show that capital stock again converges to the trap level of \bar{k}_{min} .

If $\theta < 1$, then $f_{min}^* > f_{max}^*$ (FIGURE 1, Panel (c)). In that case, the economy starts from k_t , wherein $k_t > k_Q > \bar{k}_{max} > \bar{k}_{min}$ (equivalently $f_t < f_Q < f_{max}^* < f_{min}^*$). Since $k_t > k_Q$, protection is already effective and the capital stock decreases on the path towards \bar{k}_{max} ; or equivalently, average productivity of capital decreases towards the level of f_{max}^* . However, since $k_Q > \bar{k}_{max}$, the value of k_t drops below k_Q before reaching \bar{k}_{max} and at this level of capital, protection becomes deactivated and leads to the minimum probability of capital survival which then triggers a shift from φ_{max} to φ_{min} . The economy will be entrapped in the new steady states level of \bar{k}_{min} (f_{min}^*) and foreign aid fails to achieve \bar{k}_{max} (f_{max}^*).

If $\theta > 1$, then $f_{max}^* > f_{min}^*$ (FIGURE 2, Panel (c)). We suppose that the economy starts from a stock of productive capital level k_t in which $k_t > k_Q > \bar{k}_{min} > \bar{k}_{max}$ (equivalently $f_k < f_Q < f_{min}^* < f_{max}^*$). Again, since $k_t > k_Q$, protection has already been effective and the capital stock decreases on a path towards \bar{k}_{max} , however it drops below k_Q before reaching \bar{k}_{max} , which then triggers a shift from φ_{max} to φ_{min} . The economy will be entrapped in the steady-states level of \bar{k}_{min} (f_{min}^*) and never achieve \bar{k}_{max} (f_{max}^*). ■

A widely-recognized method of escaping any form of poverty trap is to inject productive capital from the outside. However, as long as the level of capital injection is below the gap, $g = k_Q - \bar{k}_{min}$, the economy is unable to maintain the level of protective capital above the threshold level and remains in the trap. When the injected capital exceeds g through a big push, the economy might be able to obtain enough production in order to invest in protective capital and maintain its level above

Q. In this case, individuals preferences play a significant role. Proposition 3 and 4 demonstrates this situation.

Proposition 3. In a CRRA-Cobb Douglas OLG model with a pulse probability function of productive capital survival, if $\theta < 1$ and $f_{min}^* < f_Q < f_{max}^*$ then an escapable poverty trap exists.

Proof: It is supposed that the initial capital stock is below \bar{k}_{min} or equivalently $f_t > f_{min}^*$ (FIGURE 1, Panel (b)). Capital stock converges towards \bar{k}_{min} on the path of φ_{min} . At this inferior steady-state level, any external capital injection below the gap of $g = k_Q - \bar{k}_{min}$ is futile as the economy converges back to the steady-state level of \bar{k}_{min} . However, an external capital injection above the gap level of g activates the higher protection for productive capital and shifts the economy to a new path of φ_{max} towards the superior steady-state level of \bar{k}_{max} . ■

An important point here is the role of intertemporal elasticity of substitution ($1/\theta$). Since $\theta < 1$, substitution effects dominate income effects; thus, when the economy shifts to the new path of φ_{max} , individuals tend to save more and this helps further the accumulation of productive capital towards a superior steady-state of \bar{k}_{max} . In contrast, Proposition 4 demonstrates a situation in which income effects dominate substitution effects. As a result, when the economy shifts to the new path, individuals increase their early consumption such that the superior path cannot be sustained and the economy arrives at an ongoing alternation between the two paths.

Proposition 4. In a CRRA-Cobb Douglas OLG model with a pulse probability function of productive capital survival, if $\theta > 1$ and $f_{min}^* < f_Q < f_{max}^*$, then an alternating poverty trap exists.

Proof: It is supposed that the initial capital stock is below \bar{k}_{min} or $f_t > f_{min}^*$ (FIGURE 2, Panel (b)). Capital stock converges towards \bar{k}_{min} on the lower path of φ_{min} . However, before reaching \bar{k}_{min} , it strikes the threshold level of k_Q , or equivalently f_Q , which activates capital protection and the economy shifts to the upper path of φ_{max} . On the new path, the current capital stock is above the new steady-state level of \bar{k}_{max} or equivalently $f_Q > f_{max}^*$. Therefore, the economy starts to consume more and save less towards a new steady-state level which is lower than the current amount. Consequently, the capital stock level drops below the threshold level of \bar{k}_Q and again the economy shifts back to φ_{min} . In this situation, the economy will not reach either of the two steady-state levels, but alternate about \bar{k}_Q . ■

The alternating poverty traps might appear unrealistic as the economy alternates between maximum and minimum probability of capital survival. This situation stems from the strong assumption of the pulse probability function. In actuality, the probability of capital survival rises gradually as protective capital accumulates. In section D, a general probability function is employed that substitutes the alternating poverty trap with a steady-state between the two extreme levels of \bar{k}_{min} and \bar{k}_{max} .

Using CRRA utility, this study emphasizes on individuals' preferences and thereby reaches unique results. It is noteworthy that in such situations the literature of growth economics usually considers a logarithmic utility function which is the case of $\theta = 1$; however, as FIGURE 3 illustrates such an assumption will result in disappearance of traps and a zero investment level in protective capital as the optimal policy.

D. General Form Probability Function

In this section, the conclusions in C are extended by employing a general probability function that satisfies assumptions 2, 3, and 4. For the new probability function, two

thresholds, Q_a and Q_b , are assumed as the lower and upper limits for protective capital to be effective in incremental enhancement of probability capital survival. The probability of productive capital survival is defined as

$$\pi_{(q)} = \begin{cases} P_{max} & \text{if } q > Q_b, \\ P_{(q)} & \text{if } Q_a \leq q \leq Q_b, \\ P_{min} & \text{if } q < Q_a, \end{cases}$$

where $\pi_{(q)}$ is continuous and increasing in q ; that is $P'_{(q)} \geq 0$, $P_{(Q_a)} = P_{min}$ and $P_{(Q_b)} = P_{max}$.

Similar to the case of pulse probability function, the minimum levels of productive capital that are required to raise the level of protective capital to the threshold levels of Q_a and Q_b are defined as

$$k_{Q_a} = (Q_a / (\tau(1 - \alpha)A))^{1/\alpha}, \quad \text{and} \quad k_{Q_b} = (Q_b / (\tau(1 - \alpha)A))^{1/\alpha}.$$

Again, similar to f_Q :

$$f_a \equiv A \cdot k_{Q_a}^{\alpha-1} = A^{1/\alpha} \cdot (Q_a / (\tau(1 - \alpha)A))^{(1-\alpha)/\alpha} \quad \text{and}$$

$$f_b \equiv A \cdot k_{Q_b}^{\alpha-1} = A^{1/\alpha} \cdot (Q_b / (\tau(1 - \alpha)A))^{(1-\alpha)/\alpha}.$$

This is the equivalent value of f_t that is associated with the threshold levels Q_a and Q_b .

Depending upon the values of Q_a and Q_b , relative to f_{max}^* and f_{min}^* and for the cases of θ being less than, or greater than one, an economy can be in no trap, in an escapable, or in an inescapable traps or in a semi-trap.

<FIGURE 4>

The introduction of a general probability function reaffirms most of the results of the less-complicated pulse probability. Therefore, the results of Proposition 1 of no-trap conditions can simply be extended to the conditions of $f_a > f_{min}^*$ and $f_b > f_{max}^*$ in the case of $\theta < 1$ (FIGURE 4, Panel (a)) and $f_a > f_b > f_{max}^*$ in the case of $\theta > 1$ (FIGURE 5, Panel (a)), respectively. Additionally, the results of Proposition 2 regarding an inescapable trap is extendable to the conditions of $f_a < f_{min}^*$ and $f_b < f_{max}^*$ in the case of $\theta < 1$ (FIGURE 4, Panel (c)) and $f_b < f_a < f_{min}^*$ in the case of $\theta > 1$ (FIGURE 5, Panel (c)).

<FIGURE 5>

In a more subtle way, the escapable trap of Proposition 3 can be extended to a general probability model when $f_a > f_{min}^*$ and $f_b < f_{max}^*$ in the case of $\theta < 1$ (FIGURE 4, Panel (b)). An economy with a low stock of capital accumulates the two types of capital and thereby average capital productivity declines on the curve of φ_{min} . However, a minimum threshold level triggers higher protection at f_a before reaching f_{min}^* . As a result, the economy shifts to a path below φ_{min} and above φ_{max} . This new path has a lower steady-state level of f^* that is lower than the current level of f_t . The economy continues accumulating capital and the path of φ incrementally shifts downward until it reaches a steady-state level of $f_{inferior}^*$ on a path which is called φ_{Inf} . In case of a minor capital injection into the economy, the value of f_t drops below $f_{inferior}^*$ on a new path which is below φ_{Inf} . The new steady-state level of this path is greater than f_t . This new economy with incrementally higher capital

stock relative to the trap and lower average capital productivity relative to $f_{inferior}^*$ would increase early consumption and eventually move back to $f_{inferior}^*$.

While an incremental capital injection would be futile in this economy, a big push could change the situation. In the case of a capital injection large enough to raise the value of k above k_{Q_b} , which would force the level of f below f_b , the level of capital protection would attain the maximum possible level and the path of the economy would shift to φ_{max} . However, in this situation, $f_t > f_b > f_{max}^*$. Thus, the economy would consume more and invest less towards f_{max}^* . However, before reaching f_{max}^* , the value f_t would drop below f_b and the economy would shift incrementally upward to a new path of φ . Finally, it would reach a superior steady-state, $f_{superior}^*$, which is located on φ_{sup} . This is a new steady-state level and if the value of f_t marginally exceeds $f_{superior}^*$, f_t will return to this equilibrium. The fact that a minimal capital injection is unable to release the economy from the inferior equilibrium indicates that this is a trap; meanwhile, the reality that a big push would lead the economy to a superior steady-state level justifies the oxymoron of 'escapable trap'.

As mentioned above, if the value of f_t goes incrementally beyond $f_{superior}^*$ or drops incrementally below $f_{inferior}^*$, then the value of f_t would return to the relevant stable equilibrium, and such a movement indicates that there is another equilibrium, $f_{superior}^* \leq f_{saddle}^* \leq f_{inferior}^*$, above which the value of f_t increases and below which the value of f_t decreases. This new equilibrium is unstable and is a result of the intersection of line L with the path of φ_{saddle} .

The conditions of $f_a > f_{min}^*$ and $f_b < f_{max}^*$ are sufficient conditions for having an escapable trap. In fact, the necessary conditions are less restrictive and if $f_a > f_{saddle}^*$ and $f_b < f_{saddle}^*$ then the economy has an escapable trap with two equilibria. Notably, if $f_{max}^* \leq f_b < f_{saddle}^*$, then $f_{max}^* = f_{superior}^*$. In turn, if $f_{saddle}^* \leq f_a < f_{min}^*$, then $f_{min}^* = f_{inferior}^*$. If both conditions simultaneously hold, then f_{saddle}^* would be the third stable equilibrium.

The counterparty of an escapable trap when $\theta > 1$ is an economy with a single equilibrium between the two extreme equilibria which is called a 'semi-trap'. Although a semi-trap has an edge over the inferior steady-state, f_{min}^* , with higher consumption and capital levels, it is still dominated by the superior steady-state, f_{max}^* . Therefore, the semi-trap in the case of general probability function is more intuitive than the alternating trap in the case of pulse probability function.

Proposition 5. In a CRRA-Cobb Douglas OLG model with a general probability function of productive capital survival, if $\theta > 1$ and $f_a > f_{max}^* > f_b$ then there exists a unique steady-state (semi-trap), f_{ST}^* , where $f_{min}^* \leq f_{ST}^* \leq f_{max}^*$.

Proof: As $f_a > f_{max}^* > f_{min}^*$, the economy keeps capital accumulation towards f_{min}^* on φ_{min} and higher protection is activated as the average capital productivity falls below f_a . Therefore, the curve of φ shifts incrementally upward and since line L is increasing, the new steady-state (intersection of φ and L) is larger than the previous one, $f_{new}^* \geq f_{min}^*$. In turn, the capital accumulation process ensures that the new value of f_t is smaller than the previous one. Thus, the gap between f_t and the steady-state level decreases. As this process repeats, the gap approaches to zero and f_t reaches its steady-state level, f_1^* .

If the value of f_t drops below f_{min}^* , the new path φ would be lower than φ_{min} . Then either the new f_t is located on the line L , which means the economy obtains multiple equilibria on the line L , or its value is lower than the steady state level and will return to the value of f_1^* .

On the other hand, if the economy starts with a large capital stock and the value of f_t is less than f_{max}^* , then, accordingly, same arguments is valid for a decreasing gap process that guarantees extending f_t to its steady-state, f_2^* .

Again, if f_t goes beyond f_{max}^* , the new path φ would be higher than φ_{max} . Then, either the new f_t is located on the line L which means the economy obtains multiple equilibria on the line L or its value is above the steady-state level and will revert to the value of f_2^* .

In this economy either there is a unique equilibrium, $f_1^* = f_2^*$, or infinite equilibria on the line L between f_1^* and f_2^* . ■

5. POLICY DISCUSSION

One of the conclusions of the present paper is that lack of protective capital can be an explanatory factor for the fact that a large amount of the foreign aid awards to LDCs or sovereign loans to HIPC's fails to bring about the expected economic development. In fact, the injection of productive capital into these nations can be ineffective because the productive capital will be wasted if the required level of protective capital has not yet been attained. Therefore, focusing on 'Productive Capital Waste Management' (PCWM) is the direction recommended here.

Two types of policies regarding PCWM are 'injecting protective capital' and/or 'funding protective capital through higher wage tax rates'. A protective capital injection is an external policy that can be adopted by international institutions such as the World Bank and the IMF. Relative to the current policy of a productive capital injection, the protective capital injection is a more direct treatment, focusing on the source of the problem, which is capital waste. In contrast, raising wage tax rates is an internal policy which can be implemented by governments.

Protective Capital Injection: In a country with a developed fiscal system, a portion of the wage of young individuals is taken by the government and is invested as protective capital. However, any obstacle that interrupts this process, such as lack of an effective taxation system, misallocation of resources due to corruption, or investment in productive rather than protective capital, can entrap the economy in poverty. These obstacles make any productive capital injection ineffective due to a high possibility of capital waste. Therefore, based on the proposed model, an external injection of protective capital is preferred over productive capital.

Nevertheless, an external aid system that aims at raising the stock of protective capital to a level above the threshold can bypass the obstacles and draw the economy into a new world of high probability of capital survival. Nonetheless, the unfortunate result of the proposed model is that the injection of protective or productive capital would be effective only in a very specific situation: when $\theta < 1$ and $f_{min}^* > f_Q > f_{max}^*$. It is worth to notice that literature of economics often consider a value of $\theta > 3$. As previously mentioned, in the case of inescapable traps, capital injection is ineffective as higher capital would be consumed and have no impact on the steady-state level.

Easterly (2009) conducts a comprehensive literature review on the effectiveness of Western aid effort towards Africa. His survey contrasts 'transformational' approach to 'marginal' approach. In order to distinguish the two approaches, he considers the ambition or goal of each approach. The goal of transformational approach is a large permanent gain in an economic indicator or a number of social indicators, at the aggregate level. In a marginal approach, the ambition of the aid program is to solve a very specific problem of a target population.

The literature reports disappointing results from transformational approach, however, evidences some successes for marginal approach. It seems that there is a link between the transformational / marginal approaches and the two concepts of protective and productive capital. In fact, most of the transformational aids are focused on injecting productive capital into a poor society opposite to many of marginal projects such as health-related campaigns which are in nature protective. As a result, the ineffectiveness of transformation approach can be interpreted as the failure of productive capital injection policy, and the limited success of marginal approach can be seen as empirical potential for research on protective capital.

Increase in Wage Tax Rates: A social planner can make two impacts on an economy through higher wage tax rates. First, irrespective of the value of θ , a higher tax rate indicates a steeper L , which in turn increases the steady-state levels, f_{min}^* and f_{max}^* , while φ_{min} and φ_{max} remain the same. Second, a higher tax rate increases the threshold levels f_Q or the pair of f_a and f_b .

As FIGURE 1 illustrates, when $\theta < 1$, the necessary and sufficient condition for a no-trap situation is that $f_{min}^* \leq f_Q$ or equivalently $\bar{k}_{min} \leq k_Q$. Here, a sufficient condition, $f_{min}^* = f_Q$, is considered. From equation 9, it can be concluded that in the case of pulse probability function and $P_{min} = 0$, a wage tax rate, $\tau^{Trap-free}$, exists if and only if $1 = (1 - \alpha)(1 - \tau)f_Q$. Replacing the equivalent amount for f_Q and further simplification yields

$$Q^{1-\alpha}/A = (1 - \alpha)(1 - \tau)^\alpha \tau^{1-\alpha}. \quad (13)$$

Condition 13 sets a limit for the maximum value of Q that can be supported by a wage taxation system in order to avoid a poverty trap. As a result, if the required threshold of protective capital for a shift from low to high capital survival probability is very high relative to the value of Total Factor Productivity, A , then no wage tax rate in the range between zero and one exists to release the economy from the trap.

<FIGURE 6>

FIGURE 6 depicts the values of the right-hand side of condition 13 for the ranges of $\alpha \in (0.3, 0.8)$ and $\tau \in (0, 1)$. Based on this FIGURE, the right-hand side varies in a range between 0 and 0.3. For example when $\alpha = 0.3$, a wage tax rate of approximately 0.75 would yield the highest value for $\frac{Q^{1-\alpha}}{A}$, which is 0.3. Therefore, if Q is located in the range of $Q \leq (0.3A)^{1/1-\alpha} = 0.18A^{1.43}$, there will be a tax rate which allows the economy escape from a vicious cycle of poverty. This potential reflects the importance of Total Factor Productivity (TFP) improvement in poverty reduction. In fact, as A increases one percent, the taxation system can support a 1.43 percent higher value for Q .

One of the results of the argument above is that the wage tax system cannot support relatively high values of Q in nations with low TFP levels. Therefore, enhancement of productivity would assist poverty reduction programs from two directions: first, higher productivity implies higher income and therefore, more investment in protective capital level; and, second, higher productivity enables the economy to support higher threshold level of Q through wage tax. Therefore, any policy that would raise productivity is recommended for an entrapped economy in poverty.

On the other hand, a high tax rate reduces the individuals utility of current consumption. Another problem is to find a tax rate that maximizes individuals' utility over both periods of their lives. This tax rate, τ^* , is an optimal rate that maximizes the following problem when the optimal choices of individuals are given:

$$\max_{\tau} V_{(\tau)} = V(c_t^*, c_{t+1}^*),$$

where V is an intertemporal welfare function and c_t^* and c_{t+1}^* are individual's optimal choice, from Euler's equation and budget constraints.

As a result, a social planner faces two separate questions that might arrive at a dilemma: whether to select a tax rate that maximizes the utility of today's generation over its lifespan and remain in a trap forever, or once adopt a higher tax rate which releases the economy from the trap but at the cost of lower utility for the present generation. This dilemma appears more problematic in a democracy in which the future generation has no voting rights and the present generation is reluctant to sacrifice its early consumption. This poverty trap model once again reiterates that development requires a 'big push', however, in protection rather than production.

5. CONCLUDING REMARKS

An extensive body of knowledge exists regarding the poverty trap as a self-reinforcing mechanism in which a free market fails to develop. The present work provides theoretical explanations for the situations wherein not only free markets fail but also interventions, internal or external, are partially or completely ineffective in poverty reduction. Two factors contribute to these striking results: first, the concept of protection of productive capital; and second, individuals' preferences that shape their consumption choice in reaction to any rise in the expected rate of return of investments.

It is worth to notice that as Guvenen (2006) provides evidence, we can consider heterogeneity in intertemporal elasticity of substitution and assume low elasticity (high θ) for the poor versus high elasticity (low θ) for the rich. One of the results of this study is to shed light on the contradicting historical experiences on foreign aid. The success of Marshal Plan for post-WWII Western Europe and the failure of foreign aid to sub-Saharan Africa can be attributed to escapable or inescapable traps resulting from differences in elasticity of substitution.

This paper is an initial step towards recognition of the role of protective capital in development economics. Further theoretical work is required to study the process of protective capital creation and the role of political agents. How political incentives might result in underinvestment in protection and what are the interactions between protective capital accumulation and institutions?

The introduction of thresholds of protective capital in the probability functions accounts for a number of our results. It is reasonable to relax this assumption using continuous probability functions.

Further work is also required to examine the empirical relevance of inescapable and escapable poverty traps. If our view of underdevelopment is valid then there should be a relationship between long-term stagnation and LDCs' reactions to foreign aid. Another quantitative exercise can focus on the calibration of the low and high steady state equilibria to assess the theory.

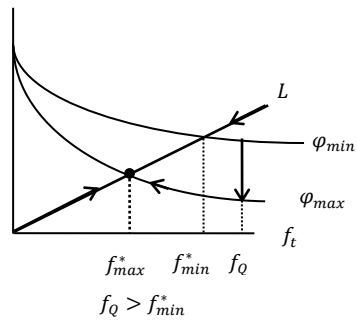
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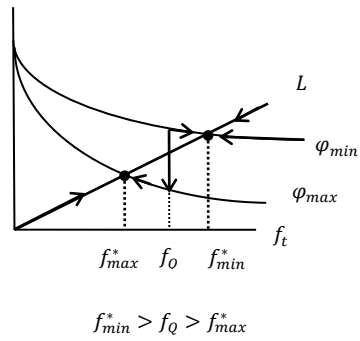
APPENDIX

FIGURE 1

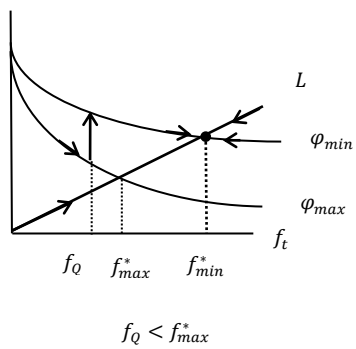
The steady-state level for a pulse probability function of productive capital survival when $\theta < 1$



(a) No Trap



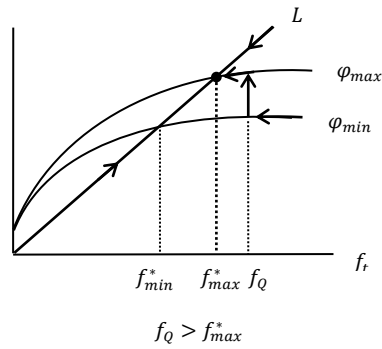
(b) With Escapable Trap



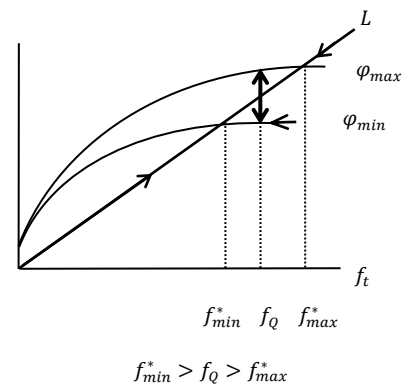
(c) With Inescapable Trap

FIGURE 2

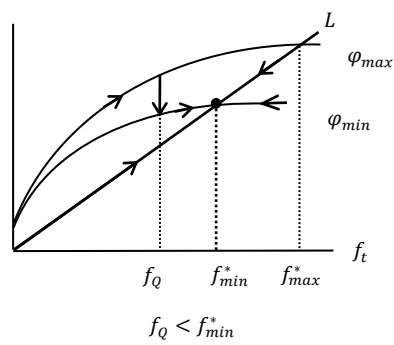
The steady-state level for a pulse probability function of productive capital survival when $\theta > 1$



(a) No Trap



(b) Alternating Inescapable Trap



(c) Inescapable Trap

FIGURE 3

The steady-state level for a pulse probability function of productive capital survival when $\theta = 1$

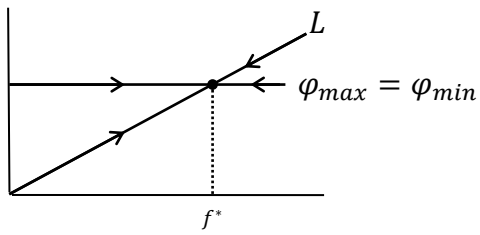
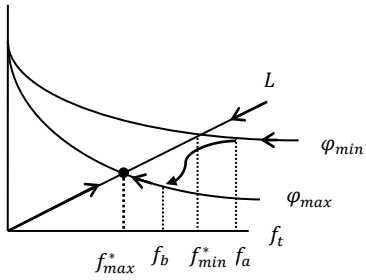


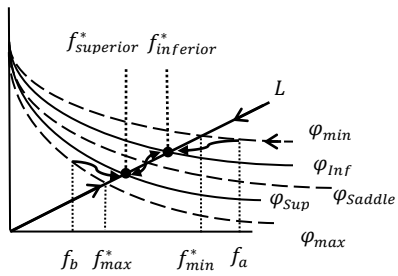
FIGURE 4

The steady-state level for a general probability function of productive capital survival when $\theta < 1$



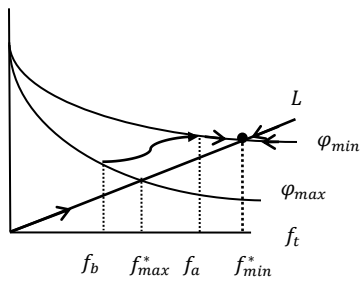
$$f_a > f_{\min}^* \text{ and } f_b > f_{\max}^*$$

(a) No Trap



$$f_a > f_{\min}^* \text{ and } f_b < f_{\max}^*$$

(b) Escapable trap

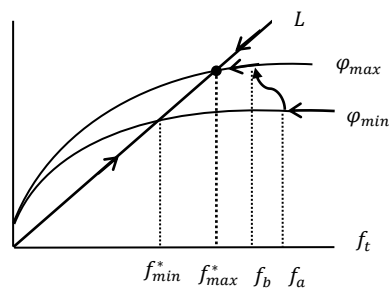


$$f_a < f_{\min}^* \text{ and } f_b < f_{\max}^*$$

(c) With Inescapable Trap

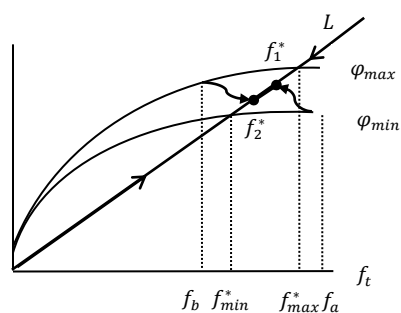
FIGURE 5

The steady-state level for a general probability function of productive capital survival when $\theta > 1$



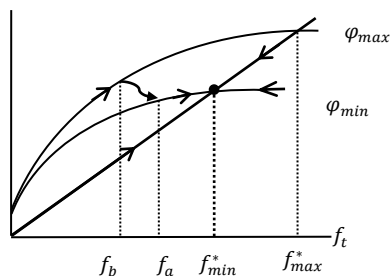
$$f_a > f_b > f_{max}^*$$

(a) No Trap



$$f_a > f_{max}^* > f_b$$

(b) Semi-Trap



$$f_b < f_a < f_{min}^*$$

(c) Inescapable Trap

FIGURE 6

Values of $\frac{Q^{1-\alpha}}{A}$ for $\alpha \in (0.3, 0.8)$ and $\tau \in (0,1)$

